SHIVAJI UNIVERSITY KOLHAPUR

Name of the Examination: **F.Y.B.Tech. Part-I Sem-I**(CBCS)

Question Bank for Mar 2022 (Summer) Examination

Name of the subject: **Engineering Mathematics-I**
Subject Code: - **71810**

1 Find the rank of matrix $\begin{bmatrix} 4 & -3 & 6 \\ 12 & -9 & 18 \\ 20 & -15 & 30 \end{bmatrix}$

2 Find the rank of matrix $\begin{bmatrix} -3 & 4 & 6 \\ 5 & -2 & -3 \\ 3 & 1 & -4 \end{bmatrix}$

3 Find the rank of matrix $\begin{bmatrix} 1 & 3 & 4 & 5 \\ 1 & 2 & 6 & 7 \\ 1 & 5 & 0 & 10 \end{bmatrix}$

4 Reduce to Normal form and find the rank of matrix $\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 2 \end{bmatrix}$

5 Reduce to Normal form and find the rank of matrix $\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$

6 Solve the equations by matrix method
$x + y + z = 3, \quad x + 2y + 3z = 4, \quad x + 4y + 9z = 6$

7 Test for consistency and if possible, Solve the equations
$2x - y + z = 9,\ 3x - y + z = 6, 4x - y + 2z = 7, -x + y - z = 4$

8 Test for consistency and if possible, Solve the equations
$2x - y + 3z = 1,\ 3x + 2y + z = 3,\ x - 4y + 5z = -1$

9 Test for consistency and if possible, Solve the equations
$x + y + z = 2,\ 2x + 2y - z = 1, 3x + 4y + z = 9$

10 Test for consistency and if possible, Solve the equations
$x + 2y - z = 3, 3x - y + 2z = 1, 2\,x - 2y + 3z = 2, x - y + z = -1$

11 Solve the following equations
$x + 2y + 3z = 0, 2x + 3y + z = 0, 4\,x + 5y + 4z = 0\,, x + 2y - 2z = 0$

12 Solve the following equations
$x + y + 2z = 0, x + 2y + 3z = 0, x + 3y + 4z, 3x + 4y + 7z = 0$

13 Solve the following equations
$x_1 + x_2 - x_3 + x_4 = 0\ , x_1 - x_2 + 2x_3 - x_4 = 0\ , 3x_1 + x_2 + x_4 = 0$

14 Solve the following equations
$$2x_1 - x_2 + 3x_3 = 0\ , 3x_1 + 2x_2 + x_3 = 0\ , x_1 - 4x_2 + 3x_3 = 0$$

15 Solve the following equations
$x_1 - x_2 + x_3 = 0\ , x_1 + 2x_2 + x_3 = 0\ , 2x_1 + x_2 + 3x_3 = 0$

16 Find the Eigen values and Eigen vector of the smallest Eigen value of the matrix
$$\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

17 Find the Eigen values and find Eigen vector of the greatest Eigen value of the matrix
$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

18 Find the Eigen values and Eigen vector of the smallest Eigen value of the matrix
$$\begin{bmatrix} 1 & -6 & 4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$$

19 Find the Eigen values and find Eigen vector of the greatest Eigen value of the matrix
$$\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

20 Find the Eigen values and find Eigen vector of the greatest Eigen value of the matrix
$$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

21 Find Eigen values of the matrix $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

22 Find Eigen values of the matrix $\begin{bmatrix} -2 & 1 & 1 \\ -11 & 4 & 5 \\ -1 & 1 & 0 \end{bmatrix}$

23 Find Eigen values of the matrix $\begin{bmatrix} 9 & -1 & 9 \\ 3 & -1 & 3 \\ -7 & 1 & -7 \end{bmatrix}$

24 Find Eigen values of the matrix $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$

25 Find Eigen values of the matrix $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

26 Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$

27 Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

28 Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 7 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 7 \end{bmatrix}$

29 Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$

30 Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

31 Simplify $\frac{(cos5\theta - isin5\theta)^2 (cos7\theta + isin7\theta)^{-3}}{(cos4\theta - isin4\theta)^9 (cos\theta + isin\theta)^5}$

32 Simplify $\frac{(cos2\theta - isin2\theta)^5 (cos3\theta + isin3\theta)^6}{(cos4\theta + isin4\theta)^7 (cos\theta - isin\theta)^8}$

33 Simplify $\left[\frac{1+Sin\left(\frac{\pi}{8}\right)+icos\left(\frac{\pi}{8}\right)}{1+Sin\left(\frac{\pi}{8}\right)-icos\left(\frac{\pi}{8}\right)}\right]^8$

34 Simplify $\left[\frac{1+cos\left(\frac{\pi}{9}\right)+isin\left(\frac{\pi}{9}\right)}{1+cos\left(\frac{\pi}{9}\right)-isin\left(\frac{\pi}{9}\right)}\right]^{18}$

35 Prove that $\left[\frac{1+sin\alpha+icos\alpha}{1+sin\alpha-icos\alpha}\right]^n = cosn\left(\frac{\pi}{2}-\alpha\right) + isinn\left(\frac{\pi}{2}-\alpha\right)$

36 Using De Moivre's Theorem Prove that
$cos4\theta = cos^4\theta - 6cos^2\theta sin^2\theta + sin^4\theta$

37 Using De Moivre's Theorem Prove that
$\frac{sin5\theta}{sin\theta} = 16cos^4\theta - 12cos^2\theta + 1$

38 Using De Moivre's Theorem Prove that
$\frac{sin6\theta}{sin2\theta} = 16cos^4\theta - 16cos^2\theta + 3$

39 Find all values of the $(\frac{1}{2} + i\frac{\sqrt{3}}{2})^{\frac{3}{4}}$

40 Solve $x^5 = 1 + i$ and find the continued product of the roots

41 Find the value of $tanhlogx\ if\ x = \sqrt{3}$

42 Solve the equation $7coshx + 8sinhx = 1$ for real values of x

43 Prove that$[\frac{1+tanhx}{1-tanhx}]^n = cosh2nx + sinh2nx$

44 If $u = logtan(\frac{\pi}{4} + \frac{\theta}{2})$ then prove that $coshu = sec\theta$

45 If $coshx = sec\theta$ Then prove $x = \log(sec\theta) + tan\theta)$

46 Using Maclaurin's series prove that
$logcox = -\frac{x^2}{2} - \frac{x^4}{12} - \frac{x^6}{45} - \ - \ - \ - \ - \ - \ - \ - \ - \ - \ -$

47 Using Maclaurin's series prove that $\log(1 + tanx) = x - \frac{x^2}{2} + \frac{2x^3}{3} - \ - \ - \ - \ - \ - \ -$

48 Expand in powers of x, e^{xsinx}

49 Expand $[\frac{1+e^x}{2e^x}]^{\frac{1}{2}}$ by using standard expansions

50 Show that $sinxsinhx = x^2 - \frac{8x^6}{6!} + \ - \ - \ - \ - \ - \ - \ -$

51 Expand in powers of $x^5 - x^4 + x^3 - x^2 - 1$ in powers of (x-1)

52 Expand $logtan\left(\frac{\pi}{4} - x\right) in\ powers\ of\ x$

53 Expand $sin\left(\frac{\pi}{6} + x\right) up\ to\ x^4\ and\ find\ \sin(30^0, 30')$

54 Expand $f(x) = \sqrt{1 + x + 2x^2}\ in\ powers\ of\ (x - 1)\ using\ Taylor's\ theorem$

55 Expand of $f(x) = x^4 - 3x^3 + 2x^2 - x + 1$ in powers of (x-3)

56 Evaluate $\lim_{x\to 0} \frac{e^{2x}-(1+x)^2}{x\log(1+x)}$

57 Evaluate $\lim_{x\to 0} \frac{x^x - x}{x-1-\log x}$

58 Evaluate $\lim_{x\to \frac{\pi}{2}} \frac{\log[x-\frac{\pi}{2}]}{\tan x}$

59 Evaluate $\lim_{x\to 1}(x^2-1)\tan(\frac{\pi x}{2})$

60 Evaluate $\lim_{x\to 0} \frac{1}{x^2} - \cot^2 x$

61 If $u = \log\left(\frac{x^2+y^2}{x.y}\right)$, then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

62 If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}-\sqrt{y}}\right)$ then prove that

i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\tan u}{2}$,

ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial y \partial x} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{4}\left[\tan^3 u - \tan u\right]$

63 Find the extreme value of the function
$f(x,y) = xy(a-x-y)$

64 Determine extreme values $f(x,y) = x^3 + y^3 - 3xy$

65 If $u = x^3 e^{\frac{-x}{y}}$ then find i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$, ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$

66 1. If $u = \frac{x+y}{1-xy}$, $v = \tan^{-1}(x) + \tan^{-1}(y)$ then find $\frac{\partial(u,v)}{\partial(x,y)}$

67 If $z = x^y$, prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$

68 If $u = f(l, m, n)$ & $l = x-y$, $m = y-z$, $n = z-x$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

69 If $u = \log\left[\frac{\sqrt{x^2+y^2}}{x+y}\right]$ then find $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$

70 If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

71 Determine extreme values $f(x,y) = x^2 + y^2 + 6x + 12$.

72 If $x = v^2 + w^2,\ y = w^2 + u^2,\ z = u^2 + v^2$ then prove that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = 16uvw$

73 If $u = \sin^{-1}\left(\frac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{x^{\frac{1}{5}}+y^{\frac{1}{5}}}\right)$, then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{400}\tan u(\tan^2 u 19)$

and $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{20}\tan u$.

74 If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

75 If x=uv and y=u/v prove that JJ'=1

76 If $X = e^v.\sec u$ and $y = e^v.\tan u$, prove that J.J' = 1

77 Use Gauss elimination method to solve

$$\mathbf{2x + y + z = 10};\ \mathbf{3x + 2y + 3z = 18};\ \mathbf{x + 4y + 9z = 16}$$

78 Use Gauss elimination Method to solve

$$x + 4y - z = -5;\ x + y - 6z = -12;\ 3x - y - z = 4$$

79 Use Gauss elimination method to solve

$$x + 3y - 2z = 5;\ 2x + y - 3z = 1,\ 3x + 2y - z = 6$$

80 Use Gauss elimination method to solve

$x + 2y + 3z = 14;\ 4x + 5y + 7z = 35;\ 3x + 3y + 4z = 21$

81 Apply Gauss-Jordan method to solve the equations

$$\mathbf{x + y + z = 9},\ \mathbf{2x - 3y + 4z = 13},\ \mathbf{3x + 4y + 5z = 40}$$

82 Apply Gauss-Jordan method to solve the equations

$x - y + 2z = 5,\quad 3x + 2y + z = 10,\quad 2x - 3y - 2z = -10$

83 Apply Gauss-Jordan method to solve the equations

$x + y + z = 5,\quad 2x + 3y + z = 10,\quad 3x - 2y + 2z = 3$

84 Apply Gauss-Jordan method to solve the equations

$x + 3y + 3z = 16,\quad x + 4y + 3z = 18,\quad x + 3y + 4z = 19$

85 Use Jacobi's iteration method to solve

$20x + y - 2z = 17,\quad 3x + 20y - z = -18,\quad 2x - 3y + 20z = 25$

86 Use Jacobi's iteration method to solve

$10x + y - z = 11.19,\quad x + 10y + z = 28.08,\quad -x + y + 10z = 35.61$

87 Use Jacobi's iteration method to solve

$15x + 2y + z = 18,\quad 2x + 20y - 3z = 19,\quad 3x - 6y + 25z = 22$

88 Use Jacobi's iteration method to solve

$4x + y + 3z = 17,\quad x + 5y + z = 14,\quad 2x - y + 8z = 12$

89 Use Gauss-seidel method to solve

$83x + 11y - 4z = 95,\quad 7x + 52y + 13z = 104,\quad 3x + 8y + 29z = 71$

90 Use Gauss-seidel method to solve

$10x + 2y + z = 9,\quad 2x + 20y - 2z = -44,\quad -2x + 3y + 10z = 22$