Seat No.	

SE - 10

Total No. of Pages : 3

F.E. (All Branches) (Semester - I) Examination, December - 2018
ENGINEERING MATHEMATICS - I
Sub. Code : 59177

Day and Date : Saturday, 08 - 12 - 2018

Time : 02.30 p.m. to 05.30 p.m.

Total Marks : 100

Instructions :
1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Use of non-programmable calculator is allowed.

SECTION - I

Q1) Attempt any three of the following : **[15]**

a) Reduce matrix A to its normal form and hence find rank if

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & 7 \end{bmatrix}$$

b) Solve the equations if they are consistent

$x + 2y - 3z = -z;\quad 3x - y + 4z = 3;\quad 6x + 5y + z = -3.$

c) Apply matrix method to solve the equations

$x + 3y - 2z = 0;\quad 2x - y + 4z = 0;\quad x - 11y + 14z = 0$

d) Find the values of λ for which following equations are consistent

$x + y + z = 1;\quad 2x + y + 4z = \lambda;\quad 4x + y + 10z = \lambda^2$

Q2) Attempt any three of the following : **[15]**

a) Examine for dependence or independence of vectors

[1 1 –1 1]; [1 –1 2 –1]; [3 1 0 1]

b) Find eigen vector for least eigen value of a matrix

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

P.T.O.

c) Find eigen values of matrices A^{-1}; A^{T}; (Adj. A) and (5A) if matrix

$$A=\begin{bmatrix}3 & 10 & 5\\ -2 & -3 & -4\\ 3 & 5 & 7\end{bmatrix}$$

d) Verify Caley-Hamilton's theorem for the matrix

$$A=\begin{bmatrix}1 & 2 & 0\\ 2 & -1 & 0\\ 0 & 0 & -1\end{bmatrix}$$

Q3) Attempt any four of the following : **[20]**

a) Prove that $\left(1+i\sqrt{3}\right)^{8}+\left(1-i\sqrt{3}\right)^{8}=(-2)^{8}$.

b) Prove that $\left[\dfrac{\sin(7\theta)}{\sin\theta}\right]=7-56\sin^{2}\theta+112\sin^{4}\theta-64\sin^{6}\theta$.

c) Solve the equation and find all the roots $x^{5}=(1+i)$.

d) Solve the equation $7\cosh x+8\sinh x=1$.

e) Prove that $\tanh^{-1}(z)=\dfrac{1}{2}\log\left(\dfrac{1+z}{1-z}\right)$.

SECTION - II

Q4) Attempt any three of the following : **[15]**

a) Expand $\log(1+\sin x)$ by Maclaurin's theorem in power of x.

b) Show that $e^{e^{x}}=e\left[1+x+x^{2}+\dfrac{5}{6}x^{3}+\dfrac{5}{8}x^{4}+\ldots\right]$.

c) Expand $2x^{3}+7x^{2}+x-6$ in powers of $(x-2)$.

d) Evaluate $\lim\limits_{x\to1}\left[\dfrac{x}{x-1}-\dfrac{1}{\log x}\right]$.

Q5) Attempt any four of the following : **[20]**

a) If $u = e^{xyz}$; find $\dfrac{\partial^3 u}{\partial x \partial y \partial z}$

b) If $z = f(x, y)$ and $x = uv$; $y = u^2 - v^2$, Prove that $2\left(u^2+v^2\right)\dfrac{\partial z}{\partial y} = u\dfrac{\partial z}{\partial u} - v\dfrac{\partial z}{\partial v}$.

c) If $y = x\cos u$ then find $\left(x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}\right)$ and $\left(x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2}\right)$.

d) If $x = r\cos\theta$, $y = r\sin\theta$; evaluate $\dfrac{\partial(x, y)}{\partial(r,\theta)}$ and $\dfrac{\partial(r,\theta)}{\partial(x, y)}$.

e) Find the maximum and minimum values of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$.

Q6) Attempt any three of the following : **[15]**

a) Solve the following system of equations by Gauss elimination method.

$3x - y + 2z = 12$, $x + 2y + 3z = 11$, $2x - 2y - z = 2$

b) Solve the following system of equations by Jacobi's iteration method (Carry out 4 iterations)

$10x + y - z = 11.19$, $x + 10y + z = 28.08$, $-x + y + 10z = 35.61$

c) Solve the following system of equations by Gauss-Seidal method (Carry out 4 iterations)

$8x - 3y + 2z = 20$, $4x + 11y - z = 33$, $6x + 3y + 12z = 35$

d) Find the largest eigen value the matrix $A = \begin{bmatrix} 2 & 3 & 2 \\ 4 & 3 & 5 \\ 3 & 2 & 9 \end{bmatrix}$ by power method

with $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ as a base vector. (Carry out 3 iterations)

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